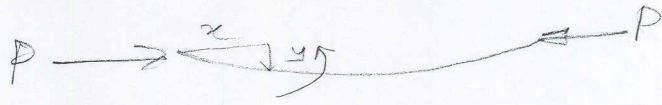


Solution (B)

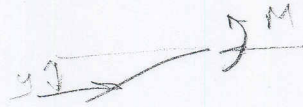
1(a)



$$\therefore EI \frac{d^2 y}{dx^2} = -Py$$

$$EI \frac{d^2 y}{dx^2} = -M$$

$$\therefore \frac{d^2 y}{dx^2} + \frac{P}{EI} y = 0$$



$$M = Py$$

$$\text{i.e. } \frac{d^2 y}{dx^2} + \alpha^2 y = 0 \quad \text{--- (1)}$$

$$\text{where } \alpha = \sqrt{\frac{P}{EI}}$$

Solution to equation (1) is

$$y = A \sin(\alpha x) + B \cos(\alpha x)$$

Boundary conditions: - when $x=0$, $y=0$

and when $x=L$, $y=0$

$$\therefore 0 = B \quad \text{and} \quad 0 = A \sin \alpha L$$

$\therefore \sin(\alpha L) = 0$ because

$A=0$ is a trivial solution

Hence $\alpha L = 0, \pi, 2\pi, \dots, n\pi \dots$

Lowest buckling load is given by

$$\alpha L = \pi$$

$$\text{i.e. } \sqrt{\frac{P_c}{EI}} L = \pi$$

$$\text{OR } P_c = \frac{\pi^2 EI}{L^2}$$

$$(b) \quad (i) \quad \hat{y} = e \left[\sec\left(\frac{\alpha l}{2}\right) - 1 \right]$$

$$\alpha = \sqrt{\frac{P}{EI}}$$

$$l = 2 \text{ m}, \quad b = 36 \text{ mm and } d = 36 \text{ mm}$$

$$I = \frac{bd^3}{12} = \frac{36^4}{12} \text{ mm}^4$$

$$\hat{y} = 20 \text{ mm}$$

$$E = 200 \times 10^9 \frac{\text{N}}{\text{m}^2}$$

$$P = 20 \times 10^3 \text{ N}$$

$$e = ?$$

$$\alpha = \sqrt{\frac{P}{EI}} = \sqrt{\frac{20000 \text{ N}}{200 \times 10^9 \text{ N/m}^2 \times \frac{36^4}{12} \text{ mm}^4}}$$

$$= \sqrt{\frac{12}{10^7 \times 36^4} \frac{\text{m}^2}{\text{mm}^4} \left[\frac{10^6 \text{ mm}^2}{\text{m}^4} \right]}$$

$$= \sqrt{\frac{1.2}{36^4} \frac{1}{\text{mm}}}$$

$$\alpha = 7.1445 \times 10^{-7} \text{ mm}^{-1}$$

$$\therefore \hat{y} = e \left[\sec\left(\frac{\alpha l}{2}\right) - 1 \right]$$

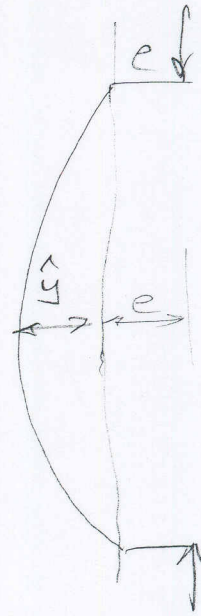
$$\text{i.e. } 20 \text{ mm} = e \left[\sec\left(\frac{7.1445 \times 10^{-7} \times 2000}{2}\right) - 1 \right]$$

$$20 \text{ mm} = e \left[\sec(0.71445) - 1 \right] = 0.50687$$

$$\text{i.e. } e = \frac{20}{0.50687} = 39.45 \text{ mm} \quad \underline{49.32 \text{ mm}}$$

25.102°

(ii) Maximum tensile and compressive stresses occur at the centre of the strut, (ie $x = \frac{L}{2}$).



$$\sigma = -\frac{P}{A} \pm \frac{P(y+e)d}{I}$$

$$\sigma_{\sigma}^{\wedge \vee} = -\frac{P}{A} \pm \frac{P(y+e) \times d}{I}$$

$$= -\frac{20 \times 10^3 \text{ N}}{36^2 \text{ mm}^2} \pm \frac{20 \times 10^3 \text{ N} (20 \text{ mm} + 49.3 \text{ mm}) \times 18 \text{ mm}}{\left(\frac{36^4}{12}\right) \text{ mm}^4}$$

$$= \left(-\frac{20 \times 10^3}{36^2} \pm \frac{20 \times 10^3 \times 69.3 \times 18 \times 12}{36^4} \right) \frac{\text{N}}{\text{mm}^2}$$

$$= (-15.432 \pm 178.241) \frac{\text{N}}{\text{mm}^2}$$

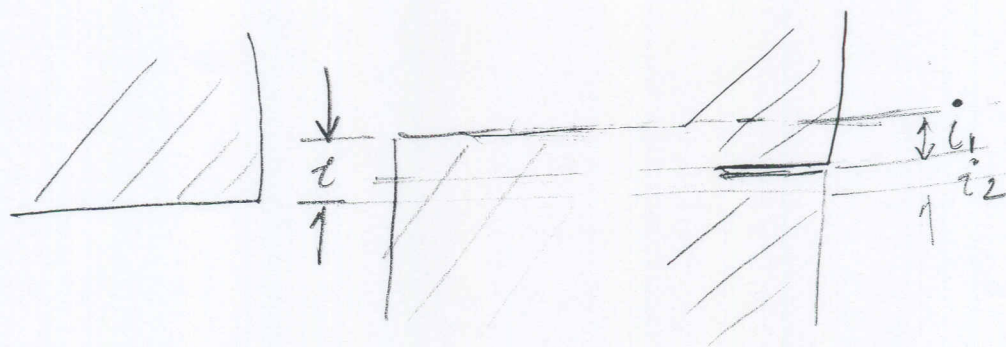
$$\therefore \text{Maximum tensile stress} = (178.241 - 15.432) \frac{\text{N}}{\text{mm}^2}$$

$$= 162.8 \frac{\text{N}}{\text{mm}^2}$$

$$\text{and maximum compressive stress} = (-178.241 - 15.432)$$

$$= -193.7 \frac{\text{N}}{\text{mm}^2}$$

Solution (2)



Before
shrink
Fitting

After
shrink
Fitting

For cylinder ①

$$\sigma_r = A_1 - \frac{B_1}{r^2}$$

$$\sigma_\theta = A_1 + \frac{B_1}{r^2}$$

$$\text{At } r = 15 \text{ mm, } \sigma_r = 0 \quad \therefore B_1 = 225 A_1$$

$$\text{At } r = 35 \text{ mm, } \sigma_r = -p$$

$$\therefore -p = A_1 - \frac{225 A_1}{1225}$$

$$\therefore A_1 = -1.225 p$$

$$\text{Thus } (\sigma_r)_1 = -1.225 p \left(1 - \frac{225}{r^2}\right)$$

$$\text{and } (\sigma_\theta)_1 = -1.225 p \left(1 + \frac{225}{r^2}\right)$$

$$\epsilon_\theta = \frac{u}{r} = \frac{1}{E} (\sigma_\theta - \nu (\sigma_r + \sigma_z)) = \frac{1}{E} (\sigma_\theta - \nu \sigma_r)$$

At the outside of cylinder ①, $r = 35 \text{ mm}$

$$\therefore \frac{-i_1}{207000} = \frac{1}{207000} (\sigma_\theta - \nu \sigma_r)$$

$$\text{i.e. } \frac{-i_1}{35} = \frac{1}{207000} \times (-1.225 p) \left(1 + \frac{225}{35^2} - \nu \left(1 - \frac{225}{35^2}\right)\right)$$

$$\text{i.e. } \frac{i_1}{35} = \frac{p}{207000} (1.45 - \nu)$$

For cylinder (2)

$$\sigma_r = A_2 - \frac{B_2}{r^2}$$

$$\sigma_\theta = A_2 + \frac{B_2}{r^2}$$

At $r = 55 \text{ mm}$, $\sigma_r = 0$, $\therefore B_2 = 3025 A_2$

At $r = 35 \text{ mm}$, $\sigma_r = -p$,

$$\therefore -p = A_2 - \frac{3025 A_2}{1225}$$

$$\text{i.e. } A_2 = \frac{1225 p}{1800}$$

and $B_2 = 3025 \times \frac{1225 p}{1800}$

Therefore, $(\sigma_r)_2 = \frac{1225 p}{1800} \left(1 - \frac{3025}{r^2}\right)$

and $(\sigma_\theta)_2 = \frac{1225 p}{1800} \left(1 + \frac{3025}{r^2}\right)$

At the inside of cylinder (2)

$$\epsilon_2 = \frac{1}{207000} (\sigma_\theta - \nu \sigma_r)$$

$$= \frac{1}{207000} \left(\frac{1225 p}{1800}\right) \left[1 + \frac{3025}{1225} - \nu \left(1 - \frac{3025}{1225}\right)\right]$$

$$= \frac{p}{207000} (3.633 - 1.469 \nu)$$

But $u_1 + u_2 = i = 0.05 \text{ mm}$

$$\therefore \frac{35p}{207000} (1.45 - \nu) + \frac{35p}{207000} (3.633 - 1.469\nu) = 0.05 \text{ mm}$$

Now, $\nu = 0.3$

$$\therefore [(1.45 - 0.3) + (3.633 - 0.441)] p = \frac{207000 \times 0.05}{35}$$

$$4.342 p = \frac{207000 \times 0.05}{35}$$

$$\therefore p = 68.11 \frac{\text{N}}{\text{mm}^2}$$

For cylinder (1)

$$\sigma_r = \underbrace{-1.225 \times 68.11}_{-83.43} \left(1 - \frac{225}{r^2}\right)$$

$$\sigma_\theta = -1.225 \times 68.11 \left(1 + \frac{225}{r^2}\right)$$

For cylinder (2)

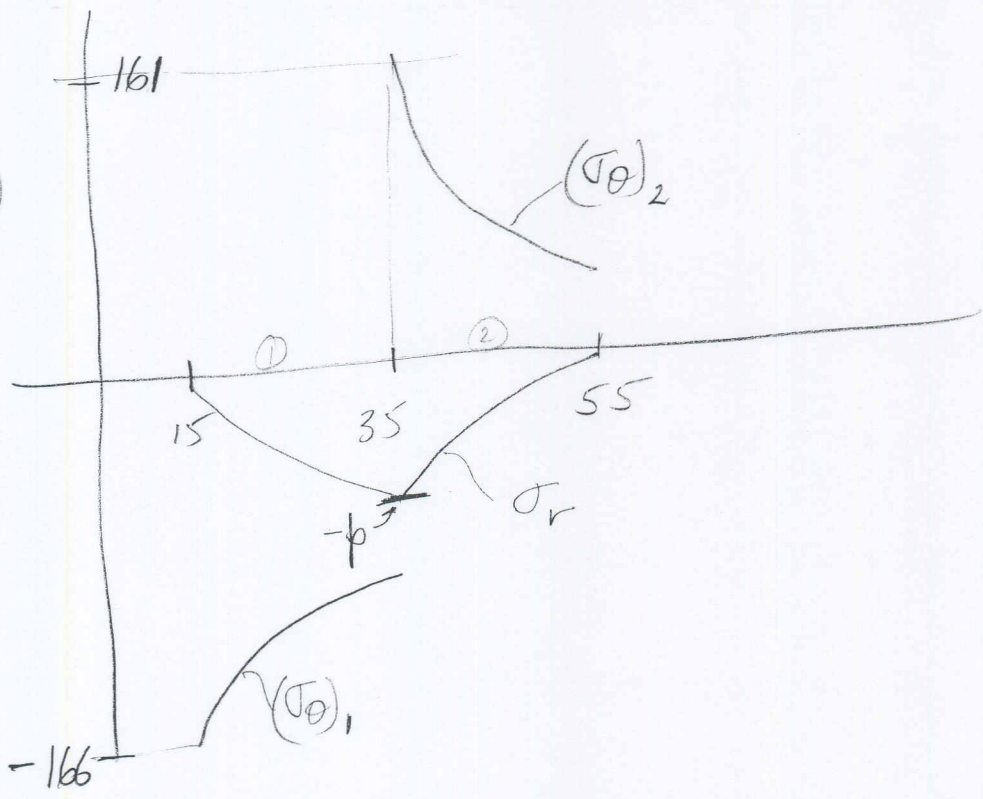
$$\sigma_r = \frac{1225}{1800} \times 68.11 \left(1 - \frac{3025}{r^2}\right)$$

$$\sigma_\theta = \frac{1225}{1800} \times 68.11 \left(1 + \frac{3025}{r^2}\right)$$

At $r = 35$, $(\sigma_\theta)_2 = 160.8 \text{ N/mm}^2$ ^{46.35}

At $r = 15$, $(\sigma_\theta)_1 = -166 \text{ N/mm}^2$

σ_{θ}
(N/mm²)

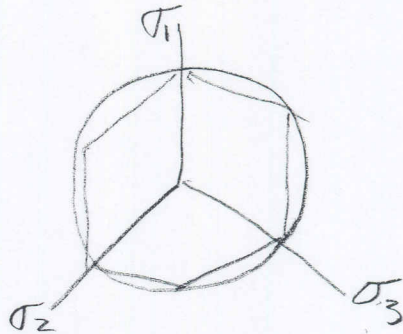
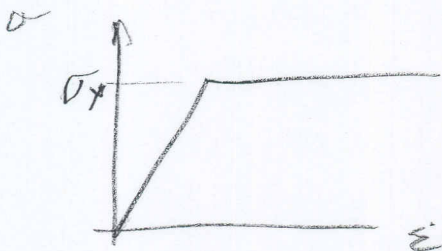


Solution (3)

(a)

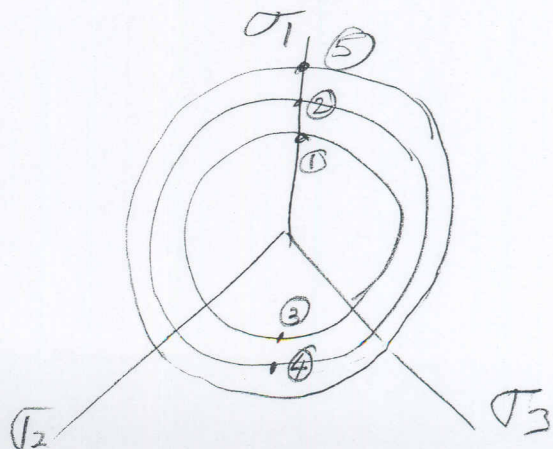
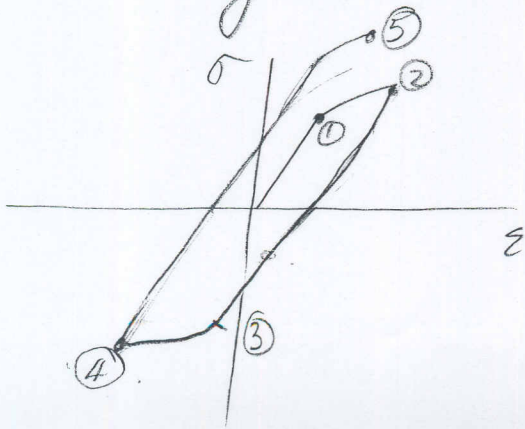
Elastic - perfectly - plastic

- no hardening
- yield locus does not change position size or shape
- gives hysteresis loops with alternating plasticity



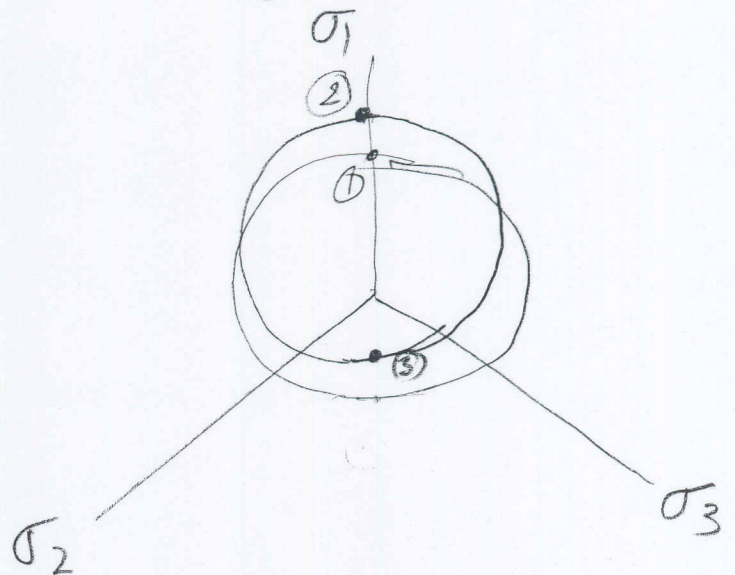
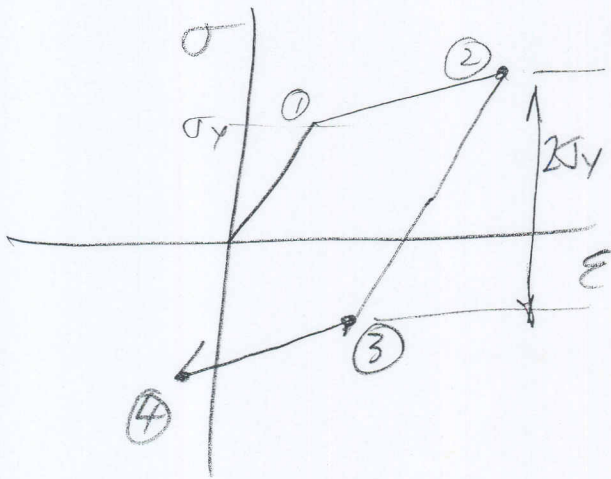
Isotropic Hardening

- plastic deformation increases yield stress
- yield surface expands isotropically with loading, reverse loading and reloading involving plastic deformation
- cyclic tension/compression, with initial yield shakes down to elastic behavior



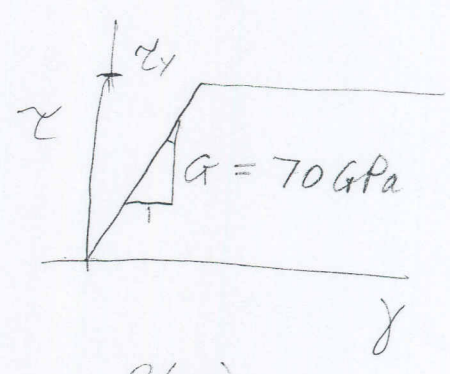
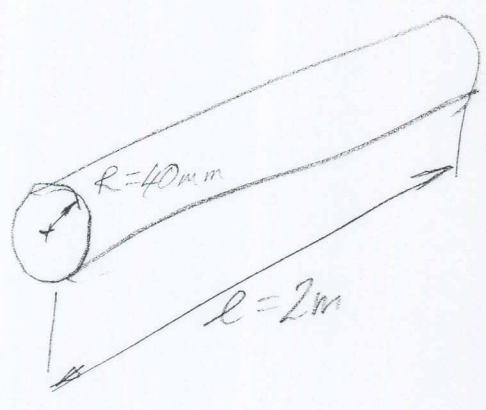
Kinematic Hardening

- assumes yield range remains constant at $2\sigma_y$
- yield surface translates in stress space with no change in shape or size
- tensile/compressive cyclic loading which initially gives yield, also gives yield for subsequent cycles.
- steady-state of alternating plastic strain sets in (hysteresis loops)



Solution

(b)

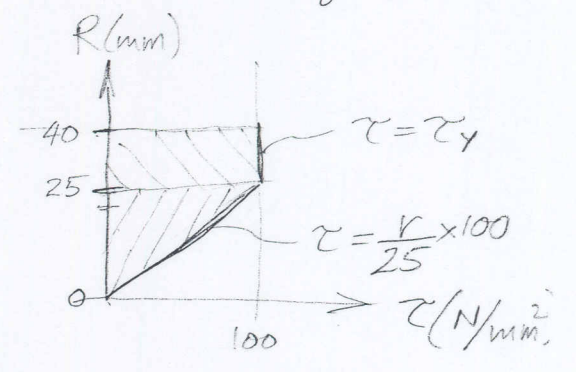


(i)

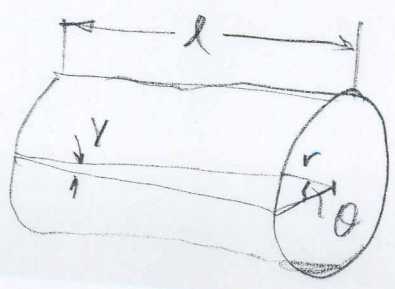
Equilibrium

$$\begin{aligned}
 T &= \int_0^{40} r \times \tau \times 2\pi r \, dr \\
 &= \int_0^{40} 2\pi \tau r^2 \, dr \\
 &= \int_0^{25} 2\pi \times \frac{r}{25} \times 100 \times r^2 \, dr + \int_{25}^{40} 2\pi \times \tau_y r^2 \, dr \\
 &= 8\pi \int_0^{25} r^3 \, dr + 200\pi \int_{25}^{40} r^2 \, dr \\
 &= 8\pi \left[\frac{r^4}{4} \right]_0^{25} + 200\pi \left[\frac{r^3}{3} \right]_{25}^{40} \\
 &= 2\pi \times 25^4 + \frac{200\pi}{3} [40^3 - 25^3] \\
 &= 2.4547 \times 10^6 + 10.1330 \times 10^6 \text{ Nmm}
 \end{aligned}$$

∴ $T = 12.588 \text{ kNm}$



Compatibility



$r\theta = \gamma l$

At $r = 25 \text{ mm}$, $\tau = \tau_y$, $\gamma = \gamma_y$

and $G = \frac{\tau_y}{\gamma_y}$, i.e., $\gamma_y = \frac{\tau_y}{G}$

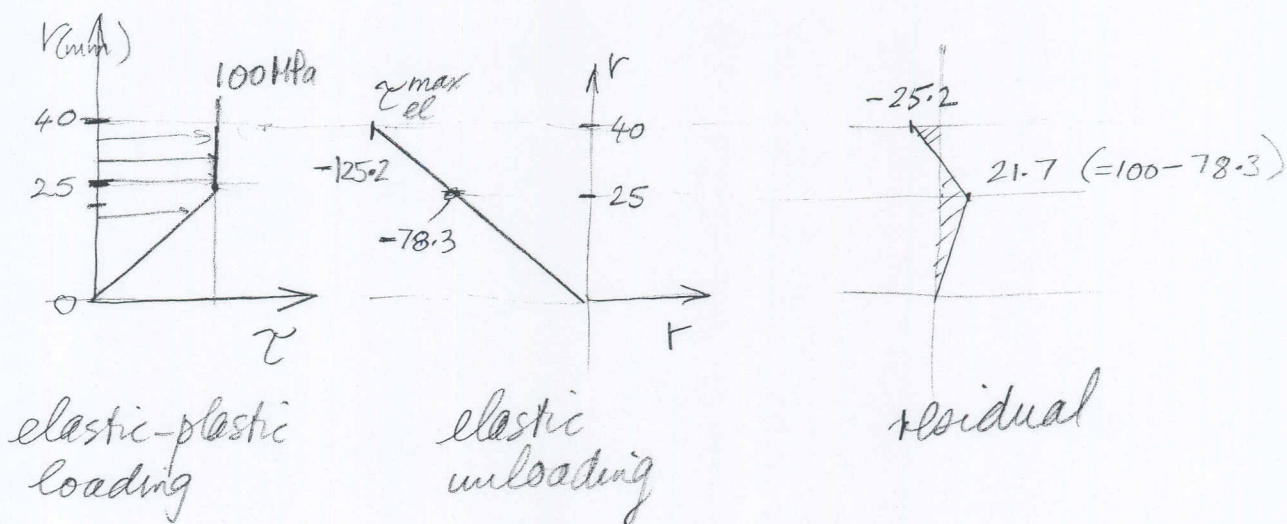
$$\therefore \gamma_y = \frac{100 \text{ N/mm}^2}{70 \times 10^3 \text{ N/mm}^2} = 1.4286 \times 10^{-3}$$

$$\therefore \theta = \frac{\gamma_y l}{r} = 1.4286 \times 10^{-3} \times \frac{2000 \text{ mm}}{25 \text{ mm}}$$

$$\text{i.e. } \theta = 0.11429 \text{ rad} \times \frac{360^\circ}{2\pi \text{ rad}}$$

$$\therefore \theta = 6.547^\circ$$

(ii)



Elastic unloading, $\tau_{el}^{\max} = \frac{Tr}{J}$, $J = \frac{\pi d^4}{32} = \frac{\pi \times 80^4 \text{ mm}^4}{32}$

$$\tau_{el}^{\max} = \frac{-12.588 \times 10^6 \text{ Nmm} \times 40 \text{ mm}}{4.02176 \times 10^6 \text{ mm}^4}$$

$$\tau_{el}^{\max} = -125.2 \text{ N/mm}^2$$

$$\therefore (\tau_{el})_{r=250} = \frac{25}{40} \times (-125.2) \frac{N}{mm^2}$$

$$= \underline{-78.3 \text{ N/mm}^2}$$

Residual angle of twist is (again) determined using the condition at interface between ^{purely} elastic and ^{elastic-} plastic behaviour, @ $r=250$

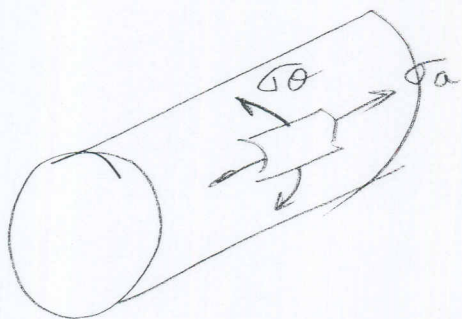
$$\gamma_{r=25} = \frac{\tau_{residual}}{G} = \frac{21.7}{70 \times 10^3} = 3.1 \times 10^{-4} \text{ rad}$$

$$\therefore \theta_{residual} = \frac{3.1 \times 10^{-4} \times 2000}{25} \times \frac{360}{2\pi} \left(\theta = \frac{\gamma l}{r} \right)$$

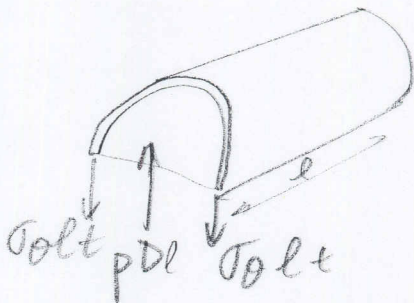
$$= \underline{1.42^\circ}$$

Solution (4)

$$(a) \quad \left. \begin{array}{l} D = 50 \text{ mm (inside)} \\ t = 1 \text{ mm} \end{array} \right\} \therefore \frac{R}{t} = 25$$



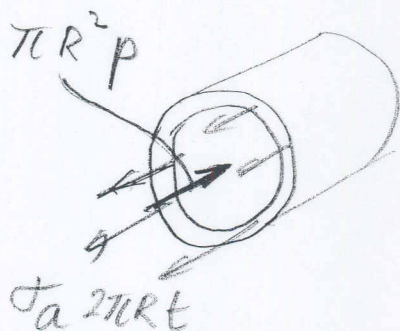
Assuming thin wall theory, -



$$2\sigma_{\theta} t = p D l$$

$$\sigma_{\theta} = \frac{p D}{2t} = \frac{p R}{t}$$

$$\underline{\sigma_{\theta} = 25p}$$



$$\sigma_a \times 2\pi R t = \pi R^2 p$$

$$\sigma_a = \frac{p R}{2t} = \underline{\underline{12.5p}}$$

$\sigma_r \approx 0$ (thin cylinder theory)

Tresca $\hat{\sigma} - \check{\sigma} \leq 250 \text{ MPa}$

$$\text{i.e. } 25p \leq 250 \text{ MPa}$$

$$\text{i.e. } \underline{\underline{p \leq 10 \text{ MPa}}}$$

von Mises $\frac{1}{\sqrt{2}} \sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2} \leq 250 \text{ MPa}$

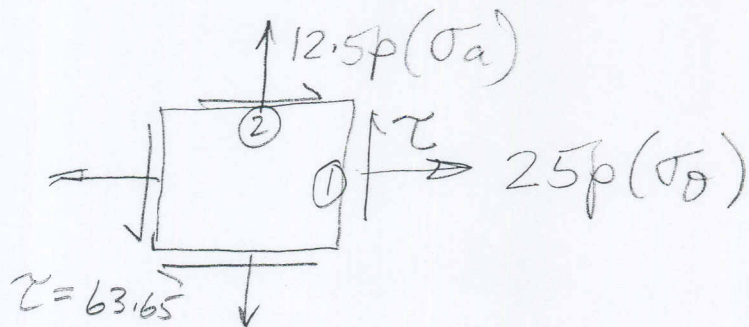
$$\frac{p}{\sqrt{2}} \sqrt{(25 - 12.5)^2 + (12.5 - 0)^2 + (0 - 25)^2} \leq 250 \text{ MPa}$$

$$\frac{p}{\sqrt{2}} \sqrt{12.5^2 + 12.5^2 + 25^2} \leq 250 \text{ MPa}$$

$$21.6506 p \leq 250 \text{ MPa}$$

$$\therefore p \leq 11.55 \text{ MPa}$$

(b)



Thin cylinder: -

$$J \approx 2\pi R t \times R^3$$

$$\text{i.e. } J \approx 2\pi R^3 t$$

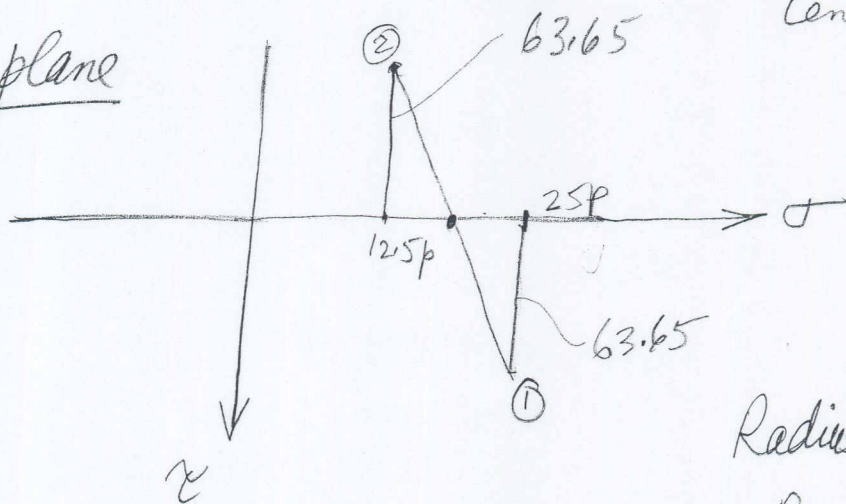
$$\frac{T}{J} = \frac{\tau}{r}$$

$$\therefore \tau = \frac{T r}{J} = \frac{0.25 \times 10^6 \text{ Nmm} \times 25 \text{ mm}}{2\pi \times 25^3 \times 1 \text{ mm}^4}$$

$$\tau = \frac{10^4}{50\pi} = 63.65 \frac{\text{N}}{\text{mm}^2}$$

Mohr's circles: -

σ_θ/σ_a plane



Centre at $\sigma = \frac{(25+12.5)p}{2}$
 $= 18.75p$

$$\text{Radius} = \sqrt{63.65^2 + (25 - 18.75)^2 p^2}$$

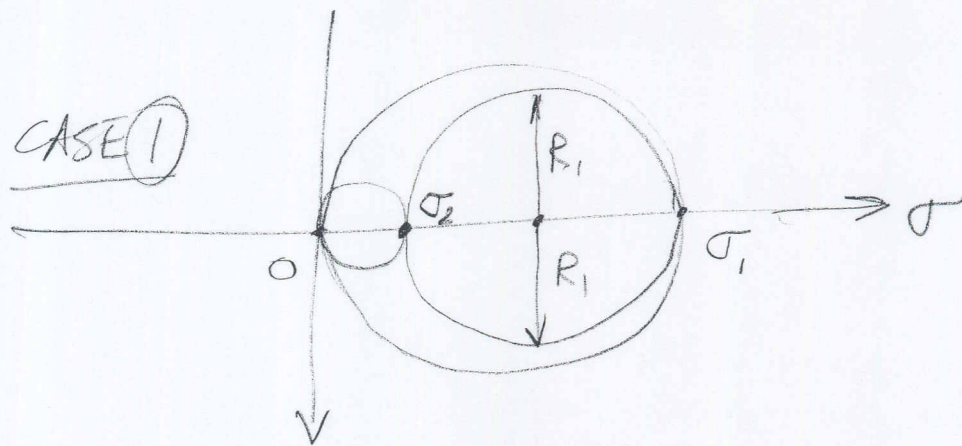
$$R_1 = \sqrt{4051 + 39.06 p^2}$$

$$\therefore \sigma_1 = 18.75 p + R_1$$

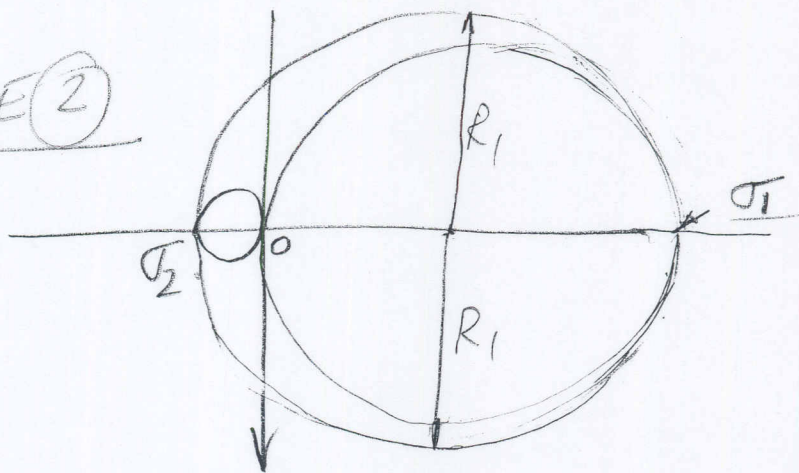
$$\text{and } \sigma_2 = 18.75 p - R_1$$

Two possible solutions

Either CASE (1)



OR CASE (2)



Case (1)

TRESCA

$$\sigma_1 - 0 = \sigma_y$$

$$\text{ie } 18.75p + R_1 - 0 = \sigma_y$$

$$\text{ie } 18.75p + \sqrt{4051 + 39.06p^2} = 250$$

$$(250 - 18.75p)^2 = 4051 + 39.06p^2$$

$$62500 - 9375p + 351.5625p^2 = 4051 + 39.06p^2$$

$$312.5p^2 - 9375p + 58449 = 0$$

$$p^2 - 30p + 187 = 0$$

$$\therefore p = \frac{30 \pm \sqrt{30^2 - 4 \times 187}}{2} = \frac{30 \pm \sqrt{900 - 748}}{2}$$

$$= \frac{30 \pm 6.16}{2} = 8.84 \text{ OR } 21.6 \text{ MPa}$$

21.16 is unrealistic since $p=10$ was obtained for p_{max} with pressure only. Therefore,

$$\underline{p = 8.84 \text{ MPa}}$$

Case (2) Tresca: -

$$\sigma_1 - \sigma_2 = 250$$

$$\text{ie } 2R_1 = 250$$

296.3

$$\therefore 4051 + 39.06p^2 = 125^2$$

$$\text{ie } \underline{p = 17.2}$$

Again, this is unrealistic because it is higher than the pressure which would cause yield on its own.

$$\therefore \underline{p = 8.84 \text{ MPa}}$$

Solution (5)

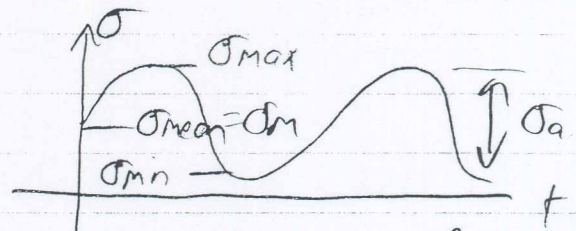
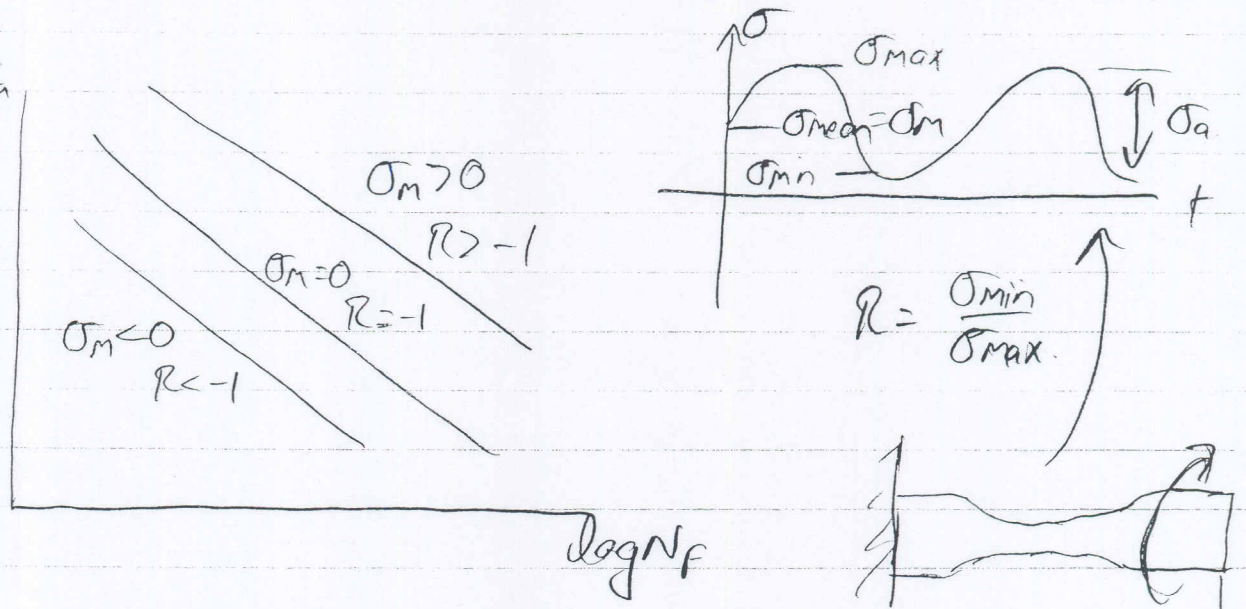
~~5~~
(a)

Total life approach

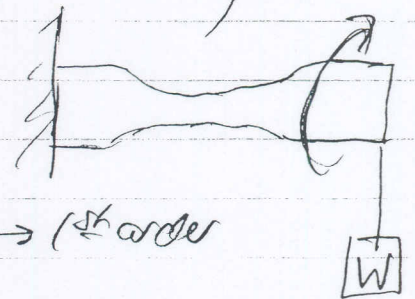
(5) Does not distinguish between crack initiation and propagation
Smooth specimens under stress-controlled conditions

log σ_a

(5)



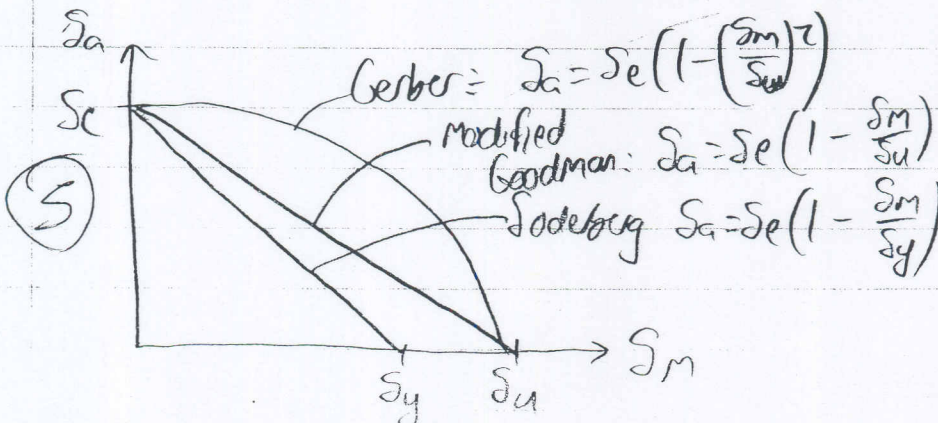
$$R = \frac{\sigma_{min}}{\sigma_{max}}$$



Effect of cyclic stress, $\sigma_a \rightarrow 1^{st}$ order

effect of mean stress on life. - 2nd order

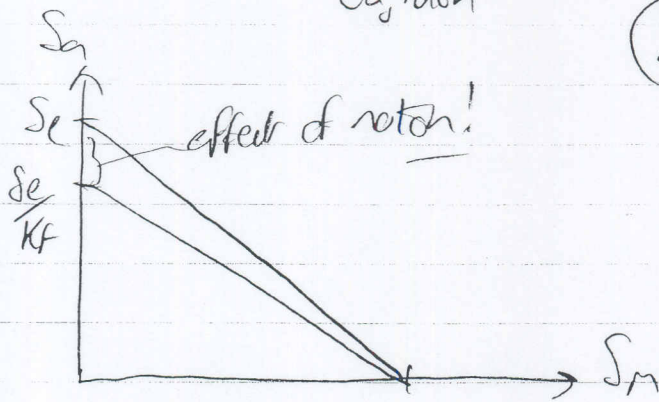
compressive \rightarrow beneficial
tensile \rightarrow detrimental



constant life lines

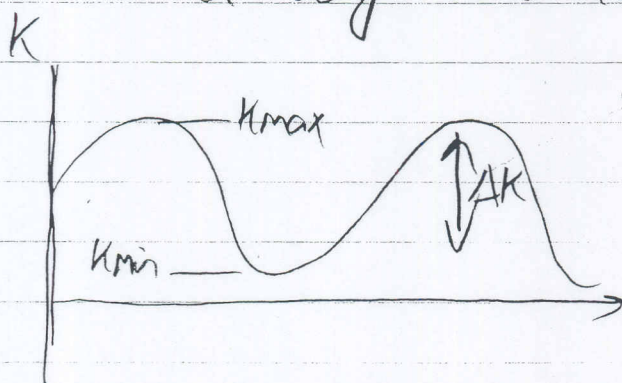
Effect of stress concentration

$$K_f = \frac{S_{a, smooth}}{S_{a, notch}}$$



Damage tolerant approach

Assumes that all structures have initial cracks and defines the fatigue life as number of cycles for crack to grow to a length which causes unstable fracture.



$$\log \frac{da}{dw}$$

(10)

Paris equation

$$\frac{da}{dw} = C(\Delta K)^m$$

$$K_{II} = K_{I}$$

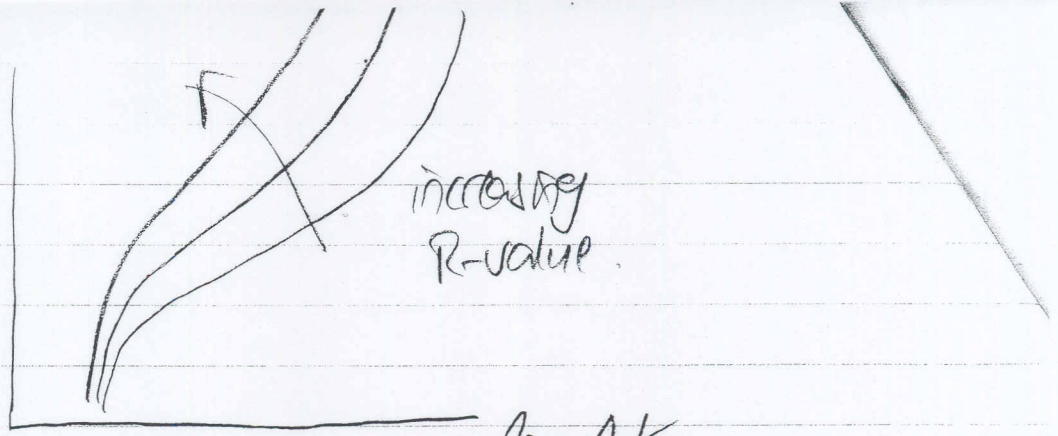
K_{II} = fracture toughness
no crack growth if $\Delta K < \Delta K_{th}$

$$\log \Delta K_{th}$$

$$\log \Delta K$$

$\log \frac{dK}{dn}$

(5)

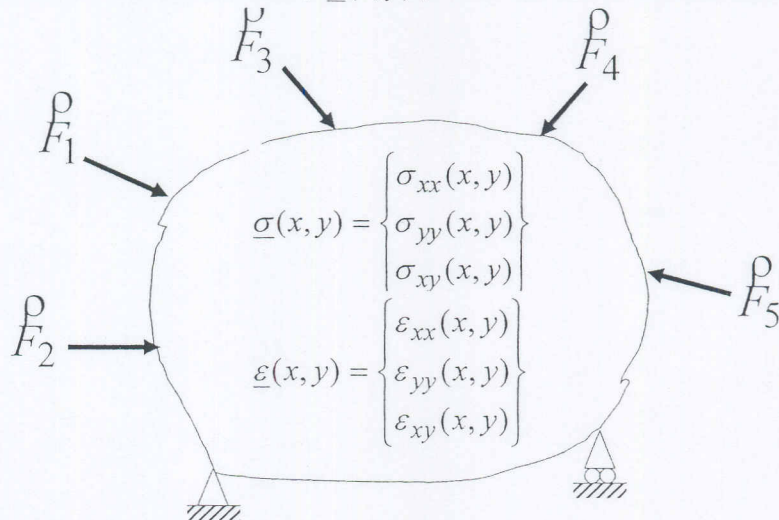


$\log AK$

Solution (5)

University of Nottingham
 School of Mechanical, Materials & Manufacturing Engrng
 Solid Mechanics 3: The Finite Element (FE) Method

(b)(i) Consider a 2D body in equilibrium under the action of a set of vector forces, \vec{F}_i , giving rise to a set of vector displacements and thence to an internal stress distribution, $\underline{\sigma}(x,y)$, and strain distribution, $\underline{\varepsilon}(x,y)$.



The principle of virtual work (PVW) says that the work done by a set of forces in equilibrium moving through a set of small, compatible displacements is zero.

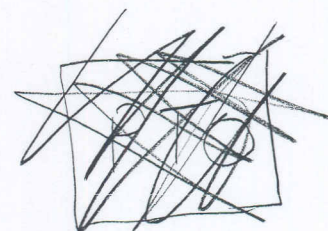
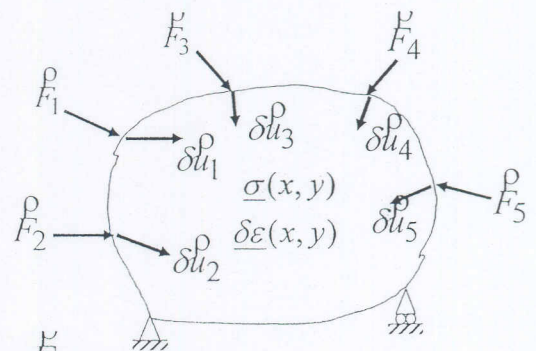
For deformable bodies, the total virtual work consists of the sum of external virtual work and internal virtual work:

$$\delta W = \delta W_{int} + \delta W_{ext}$$

where

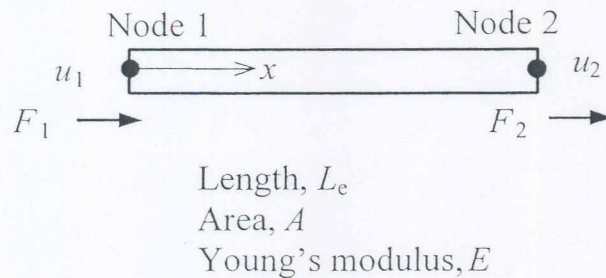
$$\delta W_{int} = - \int_V (\delta \varepsilon_{xx} \sigma_{xx} + \delta \varepsilon_{yy} \sigma_{yy} + \delta \varepsilon_{xy} \sigma_{xy}) dV$$

$$\delta W_{ext} = \delta u_1^p \cdot \vec{F}_1 + \delta u_2^p \cdot \vec{F}_2 + \delta u_3^p \cdot \vec{F}_3 + \delta u_4^p \cdot \vec{F}_4 + \delta u_5^p \cdot \vec{F}_5$$



The principle of virtual work or displacement is the foundation of structural finite element (FE) analysis as shown below. It permits the development of expressions for the stiffness matrices of a range of different structural element types. In turn this permits computational modeling of complex geometries to give approximate solutions for displacement, stress and strain distributions.

5(b)(ii) 1D example of the finite element method (FEM)

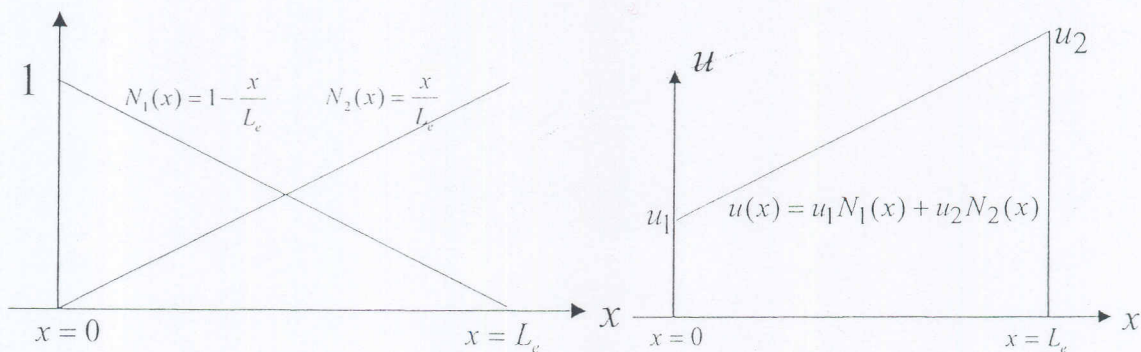


Structure is divided up into elements, connected together at nodes. The basic approximation for this element in displacement-based FE analysis is:

$$u(x) = u_1 N_1(x) + u_2 N_2(x)$$

$$= \begin{Bmatrix} N_1(x) & N_2(x) \end{Bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix}$$

where u_1, u_2 are the nodal degrees of freedom (dofs) and $N_1(x), N_2(x)$ are called the shape functions of the element.



Strain-displacement relation:

$$\varepsilon = \frac{du(x)}{dx}$$

$$\Rightarrow \varepsilon = \begin{Bmatrix} \frac{dN_1(x)}{dx} & \frac{dN_2(x)}{dx} \end{Bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \begin{Bmatrix} -\frac{1}{L_e} & \frac{1}{L_e} \end{Bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \{\underline{B}\} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \{\underline{B}\} \{\underline{u}\}$$

Stress-strain (constitutive) relation:

$$\sigma = E\varepsilon = E\{\underline{B}\} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \{\underline{D}\} \{\underline{B}\} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix}$$

Internal virtual work:

$$\begin{aligned} \delta W_{\text{int}} &= - \int_0^{L_e} \delta \underline{\varepsilon}^T \underline{\sigma} A dx \\ &= -A \int_0^{L_e} \{\delta \underline{u}\}^T \{\underline{B}\}^T E \{\underline{B}\} \{\underline{u}\} dx \\ &= -EA \{\delta \underline{u}\}^T \int_0^{L_e} \{\underline{B}\}^T \{\underline{B}\} dx \{\underline{u}\} \end{aligned}$$

External virtual work:

$$\delta W_{\text{ext}} = \delta u_1 P_1 + \delta u_2 P_2 = \{\delta \underline{u}\}^T \{\underline{P}\}$$

Principal of virtual work (PVW)

$$\begin{aligned} \delta W_{\text{ext}} + \delta W_{\text{int}} &= 0 \\ EA \{\delta \underline{u}\}^T \int_0^{L_e} \{\underline{B}\}^T \{\underline{B}\} dx \{\underline{u}\} &= \{\delta \underline{u}\}^T \{\underline{P}\} \\ \Rightarrow EA \int_0^{L_e} \{\underline{B}\}^T \{\underline{B}\} dx \{\underline{u}\} &= \{\underline{P}\} \\ \Rightarrow \{K\} \{\underline{u}\} &= \{\underline{P}\} \end{aligned}$$

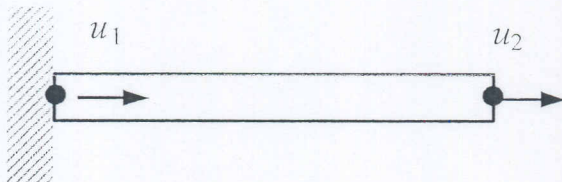
This could be provided with more words and less equations provided the process is clearly explained

This is the stiffness equation for the element of a 1D stress system.

$$\begin{aligned}
 \{K\} &= EA \int_0^{L_e} \{\underline{B}\}^T \{\underline{B}\} dx \\
 &= EA \int_0^{L_e} \begin{Bmatrix} -\frac{1}{L_e} \\ \frac{1}{L_e} \end{Bmatrix} \begin{Bmatrix} -\frac{1}{L_e} & \frac{1}{L_e} \end{Bmatrix} dx \\
 &= \frac{EA}{L_e} \begin{Bmatrix} 1 & -1 \\ -1 & 1 \end{Bmatrix} \\
 \Rightarrow \frac{EA}{L_e} \begin{Bmatrix} 1 & -1 \\ -1 & 1 \end{Bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} &= \begin{Bmatrix} P_1 \\ P_2 \end{Bmatrix}
 \end{aligned}$$

Solution of stiffness equation:

To solve the stiffness equation, need to apply boundary conditions – otherwise, elements can move as rigid bodies and we cannot solve the equations for unique values of displacement.



Suppose $u_1 = 0$, then $u_2 = \frac{P_2 L_e}{EA}$ which is the exact solution for this problem. This is because the assumed linear variation of displacement is the exact variation in this case.

Once the primary variables have been solved for, by solving the stiffness matrix, then back-substitution into the strain-displacement and stress-strain relations yield the strains and stresses in each element and consequently in the complete FE model for a multi-element analysis.

Strain:

$$\Rightarrow \varepsilon = \begin{Bmatrix} -\frac{1}{L_e} & \frac{1}{L_e} \end{Bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \{B\} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = B_2 u_2 = \frac{1}{L_e} \frac{P_2 L_e}{EA} = \frac{P_2}{EA}$$

Stress:

$$\sigma = E \{B\} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = E \varepsilon = E \frac{P_2}{EA} = \frac{P_2}{A}$$

Reactions:

$$\frac{EA}{L_e} \begin{Bmatrix} 1 & -1 \\ -1 & 1 \end{Bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \begin{Bmatrix} P_1 \\ P_2 \end{Bmatrix} = \begin{Bmatrix} R \\ P \end{Bmatrix}$$

$$u_1 = 0$$

$$\Rightarrow -\frac{EA}{L_e} u_2 = R$$

$$\Rightarrow R = -\frac{EA PL_e}{L_e EA} = -P$$

Solution requires description of the process to this point including a clear statement of the various stages. Another element type (or approach) could be used instead to illustrate the process.